Assignment 7

1. Show that the closed unit ball in $C[0,1]$ is not compact in L^1-distance.

2. Let $\{\varphi_k\}_{k=1}^{\infty}$ be an orthonormal set in $R[a,b]$. Show that it does not have any converging subsequence in L^2-distance.

3. Show every closed subset of a compact set is compact.

4. Let E be a set satisfying the finite cover property. Show that every closed subset of E also satisfies the finite intersection property.

5. Use the open cover property of a compact set to show that the image of a compact set under a continuous map is compact.

6. Show that every continuous function from (X,d) to (Y,ρ) is uniformly continuous provided X is compact.

7. Show that every continuous function from (X,d) to $\mathbb{R}$ attains its minimum and maximum provided X is compact.

8. Let $\mathcal{K}$ be the collection of all compact, convex sets in $\mathbb{R}^n$. Define

$$d(A,B) = \min\big\{\{d(x,B):\ x \in A\},\ \{d(A,y):\ y \in B\}\big\}.$$

   Verify d is a metric on $\mathcal{K}$. This metric is called the Hausdorff metric.